



## PARAMETER ESTIMATION AND UNCERTAINTY ANALYSIS OF THE VARIABLE INFILTRATION CAPACITY MODEL IN THE XITIAOXI CATCHMENT

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**ABSTRACT:** The Xitiaoxi catchment is one of the most important catchments of Tai lake basin. The frequent flood disasters have been the most serious problems in Tai lake basin due to increasing drowned areas and economic loss. In this research, VIC (Variable Infiltration Capacity) model was applied to Xitiaoxi catchment for better understanding of the flood generation process over the Tai lake basin. GLUE (Generalized Likelihood Uncertainty Estimation) method was selected to address VIC model parameter estimation and uncertainty analysis, and its contribution on model simulation uncertainties. The results indicates that: (1) VIC model performs satisfactorily in daily discharge simulation of the Xitiaoxi catchment; (2) In the seven selected parameters, the variable infiltration curve parameter B and soil depth of the first soil layer d1 were more sensitive than others; (3) Contributions of parameter uncertainty on model simulation uncertainties are relatively high.

Key Words: VIC model, GLUE, Parameter Estimation, Uncertainty Analysis

### 1. INTRODUCTION

In recent decades, the application of distributed hydrological models have been more and more popular, such as the SHE(Abbott *et al.*, 1986), SWAT (Arnold *et al.*, 1998) and VIC(Liang *et al.*, 1994). Compared with the lumped conceptual rainfall-runoff models, the distributed hydrological models can provide the spatial process of the physical phenomena occurring within the catchment. The description of such processes involves a large amount of parameters, which vary greatly across spatial and temporal scale. Therefore, Reducing the parameter space and accessing the parameter uncertainty are essential to the calibration of the distributed hydrological models.

A variety of methods is developed to deal with the parameter uncertainty, such as first-order error analysis (Melching and Yoon, 1996), the Monte Carlo method (Kao and Hong, 1996), the Bootstrap method (Li *et al.*, 2013), the Bayesian approach (Freni and Mannina, 2010) and the GLUE method (Beven and Binley, 1992). Among these methods, the GLUE method represents an extension of Bayesian or fuzzy averaging procedures to less formal likelihood or fuzzy measures (Beven *et al.*, 2000). Due to its conceptual simplicity, ease of implementation, and its flexibility with different sources of information that can be combined with different criteria (Blasone *et al.*, 2008b), the GLUE method is subsequently applied to many watersheds.

The purpose of this paper is to employ VIC model in simulating daily discharges of Xitiaoxi basin, which is one of the main source of water volume of Tai Lake. The GLUE method is adopted to address VIC model parameter estimation and uncertainty analysis, and its contribution on model simulation uncertainties.

This paper is organized as follows. A general introduction of the study area is given in the next section, followed by the brief description of VIC model and GLUE method. The section of results and discussion, including model simulation results, sensitivity and uncertainty analysis of model parameters, uncertainty estimation of discharge simulation, are presented before major conclusions of this paper.

## 2. STUDY AREA DESCRIPTION

The Xitiaoxi catchment lies in the upper reaches of the Tai Lake basin, which supplies about 27.7% of the total water volume of the Tai Lake and plays an important role in the flood management of the downstream area. The drainage area of the Xitiaoxi catchment at the outlet, which locates in the Hengtang station, is 1412 km<sup>2</sup>. The Xitiaoxi catchment has a subtropical monsoon climate. Its average annual precipitation is 1385mm and the average annual temperature is 15°C. Figure 1 shows the location of the Xitiaoxi catchment and the distribution of the Heng Tangcun gauges station and seven meteorological stations.

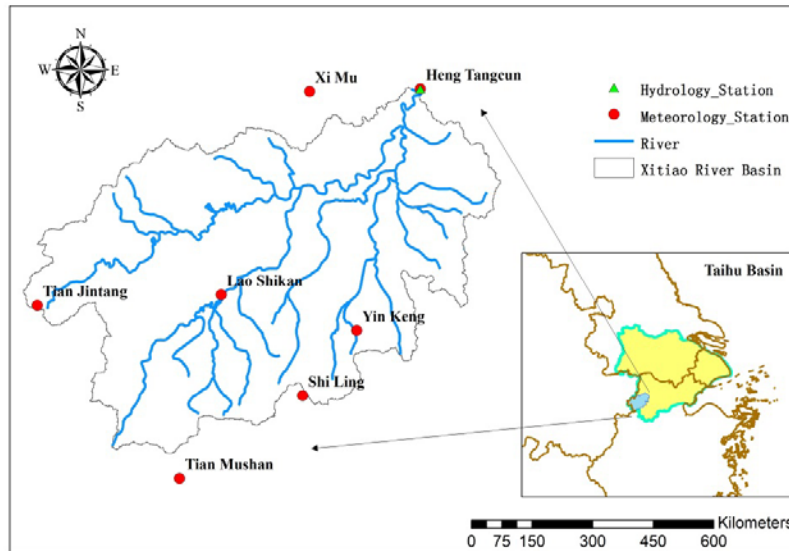


Figure 1: Map of gauge and meteorological stations and the location of Xitiaoxi catchment

## 3. METHODS

### 3.1 VIC model

The VIC model was selected to simulate the rainfall-runoff process of the Xitiaoxi catchment for better understanding of the flood generation process over the Tai lake basin, which has been demonstrated to perform especially well in simulating discharges in humid environments (Abdulla and Lettenmaier, 1997).

Based on the soil-vegetation-atmosphere (SVA) transfer scheme, Liang *et al.* (1994) proposed the VIC model to take both the water and energy balances into account. The surface of each grid cell can be horizontally described by  $N+1$  land cover titles, where  $n = 1, 2, \dots, N$  represents  $N$  different tiles of vegetation, and  $n = N+1$  represents bare soil. In the vertical direction, each grid is subdivided into canopy and three soil layers according to the three-layer VIC model (VIC-3L)(Liang *et al.*, 1996). The upper two soil layers represent the dynamic response of soil to the infiltrated rainfall. The surface runoff generating from the upper two soil layers is accounted based on the variable soil moisture capacity curve, which is described by the Xinanjiang model in order to represent the sub-grid spatial variability in soil moisture capacity (Zhao *et al.*, 1992). The bottom soil layer receives moisture from the middle layer through gravity drainage, which is regulated by a Brooks-Corey relationship (Brooks and Corey, 1988) for the unsaturated hydraulic conductivity. The runoff from the bottom soil layer is according to the drainage described by the Arno model (Franchini and Pacciani, 1991).

Generally, the characters of the VIC model can be summarized as follows (Liu *et al.*, 2011): (1) with both water balance and energy balance parameterisation; (2) with two types of run-off generation mechanisms – saturation excess run-off and infiltration excess runoff; (3) with subgrid scale soil heterogeneity; and (4) with the subgrid spatial variability of precipitation.

### 3.2 Model setup

The main input of the VIC model includes the vegetation data set, soil data set and forcing data set. The vegetation parameters were based on the University of Maryland global vegetation classifications and the Land Data Assimilation System (LDAS) information. The soil data was derived from a 1:1 000 000 Chinese soil distribution map and reclassified based on the NOAA soil parameters. The daily series of precipitation, maximum and minimum air temperature from seven meteorological stations (Figure 1) were interpolated into each grid cell using the Inverse Distance Weighting (IDW) method.

The daily observed discharges of Heng Tangcun hydrological station is used in model calibration and validation. The available data spanned from 1/1/1993 to 31/12/2000. In order to reduce uncertainty caused by state values initialization, the data from 1/1/1993 to 31/12/1994 was used as a warming-up period. Hence data from 1/1/1995 to 31/12/1997 were used for the calibration. For validating the model, data from the last three years were used.

### 3.3 GLUE Method

The GLUE method is a statistical method for quantifying the uncertainty of model predictions, which is proposed by Beven and Binley (1992). Considering the multiple sources of uncertainty in real problems and an expectation that the errors' structure will be non-stationary and complex, the less formal likelihood function is used (Beven, 1993; Beven *et al.*, 2008). In this study, GLUE method was applied in the following steps:

(1) Based on the Monte Carlo sampling from feasible parameter spaces with uniform distribution, VIC model was run with a number of parameters sets.

(2) The likelihood function, as a measure of goodness-of-fit, was chosen and calculated by the observations in every parameter set. The Nash-Suttcliffe model efficiency coefficient  $E_{ns}$  was chosen in this study, which is shown in Equation (1).

$$E_{ns} = 1 - \frac{\sum_{t=1}^N (y_t^{obs} - y_t^{sim})^2}{\sum_{t=1}^N (y_t^{obs} - y_{avg}^{obs})^2} \quad [1]$$

Where N is the length of the time series,  $y_t^{obs}$  and  $y_t^{sim}$  are the observed and simulated discharge at time  $t$ ,  $y_{avg}^{obs}$  is the mean of observed discharges.

(3) A given threshold value for behavioral parameter sets was defined (0.6). When the value of likelihood function is under the threshold, the parameter set is rejected. Then the likelihoods of the remaining parameter sets were rescaled with a cumulative sum of 1, which is defined as likelihood weight.

(4) Estimate the confidence interval of discharges at each time step for uncertainty analysis, by quantiles ranked in ascending order with likelihood weight.

## 4. RESULTS

### 4.1 Model Simulation

The GLUE method stopped at 2143 iterations. The likelihood values of these iterations were used in further analysis. The parameter of the VIC model can be estimated by the max likelihood value, which is shown in Table1.

Table 1 Parameters in the VIC model

Parameter	Description	Range	Estimated value
B	Defining the shape of the variable soil moisture capacity curve	U [0,0.4]	0.23
Ds	Fraction of Dm where non-linear baseflow begins	U[0,1]	0.45
Dmax	Maximum baseflow that can occur from the third soil layer	U[0,30]	25.79
Ws	Fraction of the maximum soil moisture of the third soil layer where non-linear baseflow occurs	U[0,1]	0.47
d1	Soil depth of the first soil layer	U[0.1,1.5]	0.32
d2	Soil depth of the second soil layer	U[0.1,1.5]	0.47
d3	Soil depth of the third soil layer	U[0.1,1.5]	0.41

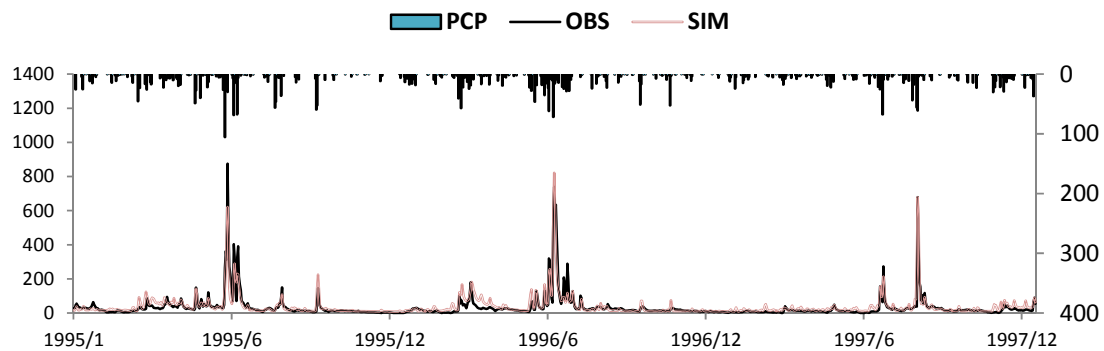
Except the Nash-Sutcliffe model efficiency coefficient, the root-mean-square error (RMSE) and mean absolute error (MAE) are also used to evaluate the model performance, which are defined in Equation 2-3. The model performance in calibration and validation period is shown in Table 2. It can be observed that the  $E_{ns}$ , RMSE and MSE are 0.82, 29.35 and 15.28 in calibration period, and 0.84, 25.61 and 16.84 in validation period respectively. The model performance was evaluated by comparison of the observations against the simulated hydrographs based on the max likelihood value from the GLUE method (Figure 2). It is shown that the simulated hydrographs showed considerable agreement with the observed hydrograph.

$$RMSE = \sqrt{\frac{\sum_{t=1}^T (y_t^{obs} - y_t^{sim})^2}{T}} \quad [2]$$

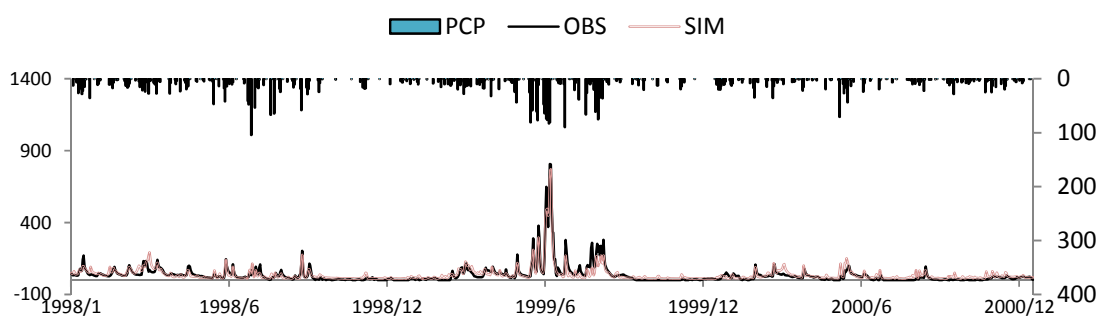
$$MSE = \frac{|y_t^{obs} - y_t^{sim}|}{T} \quad [3]$$

Table 2 Simulation results of VIC model

	$E_{Ns}$	RMSE	MAE
Calibration period (1/1/1995 to 31/12/1997)	0.82	29.35	20.08
Verification period (1/1/1999 to 31/12/1999)	0.84	25.61	16.84



(a) Calibration period

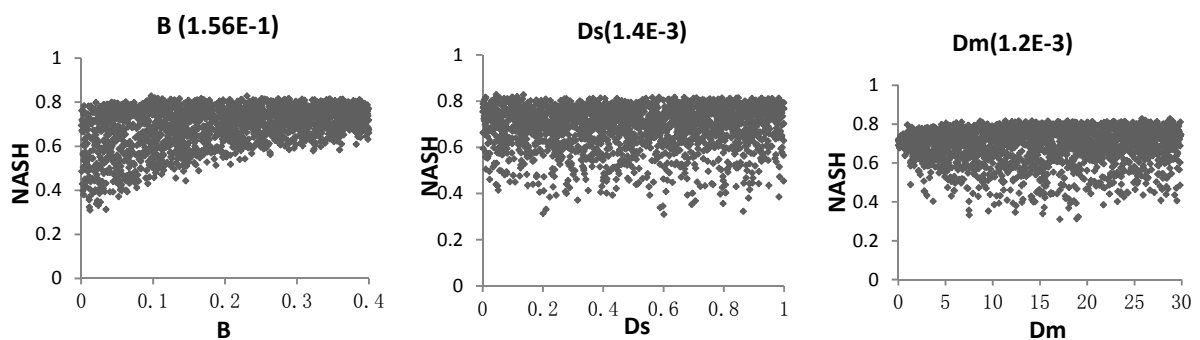


(b) Validation period

Figure 2 Simulation results of VIC model

#### 4.2 Sensitivity and uncertainty analysis of model parameters

The scatter diagram of Nash-Sutcliffe efficiency coefficient against each parameter is plotted in Fig.2. The coefficients of determination of a linear regression are also marked behind the name of the parameters. It can be observed that most of the parameters are not sensitive, except the B and d1. According to the coefficients of determination, the corresponding values are 0.16 and 0.41 for B and d1 respectively, which is much larger than those of other parameters. It may indicate that the equifinality of the VIC model is notable in Xitiaoxi catchment.



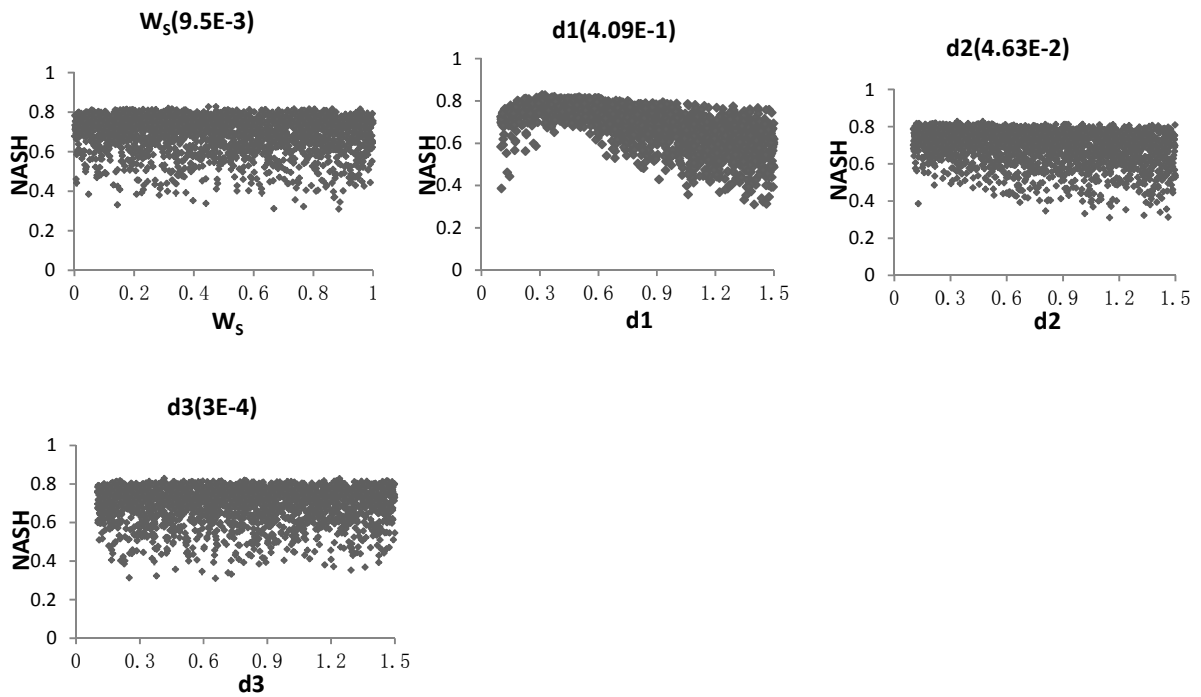


Figure 3 Scatter diagram of Nash-Sutcliffe efficiency against each parameters of the VIC model

### 4.3 Uncertainty estimation of discharge simulation

To calculate confidence intervals of model uncertainty, 1,776 discharge values for each day were obtained by running the VIC model with the behavioral parameter sets, when threshold value is 0.6. The 90% confidence intervals due to parameter uncertainty were estimated by sorting those discharge samples. The model uncertainties due to parameter errors were are plotted in Figure 4.

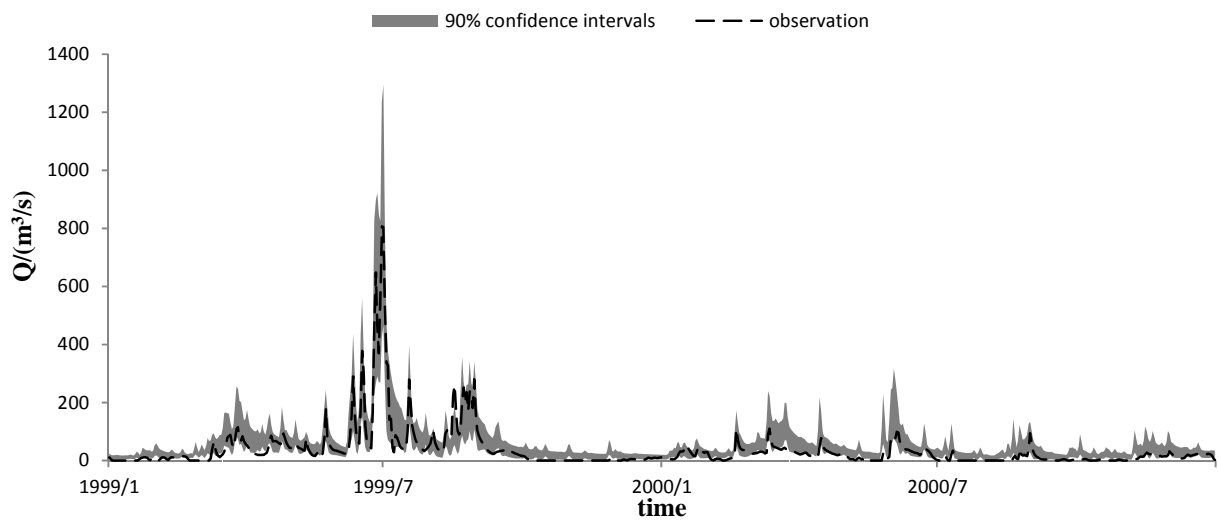


Figure 4 The 90% confidence interval for simulated discharge in Xitiaoxi catchment

It can be observed that the observations fall within the 90% confidence interval due to parameter uncertainty in most of the time, especially in simulating high flows. It indicates that the VIC model is capable for the daily discharge simulation of the Xitiaoxi catchment. However, the 90% confidence interval cannot brackets some observations in low flows, indicating that the model structure uncertainty and observation uncertainty are quit considerable in this period.

## 5. DISCUSSION AND CONCLUSION

This study demonstrates the application of the VIC model for the Xitiaoxi catchment, the upper reach of the Tai lake basin. Model parameter estimation and its uncertainty analysis, together with model simulation uncertainty resulting from parameter uncertainty are the main foci.

The VIC model shows good performance in the daily discharge simulation of the Xitiaoxi catchment with Nash-Sutcliffe model efficiency coefficients over 0.82 and 0.84 for both calibration and validation periods. The fitted hydrographs also showed considerable agreement with the observed hydrograph.

The uncertainty of the model parameters is estimated by the GLUE method. It can be observed that the equifinality of the VIC model is quit notable in the study area. According to the seven estimated parameters, only the B and d1 are sensitive. The coefficients of determination are 0.16 and 0.41 for B and d1 respectively, which is much larger than those of other parameters.

Contributions of parameter uncertainty on model simulation uncertainties are relatively high in this study. Most of the observations fall within the 90% confidence interval due to parameter uncertainty. However, the model structure uncertainty and observation uncertainty still need to be further discussed in simulating low flows.

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